

THE PERSPECTIVE-N-POINT PROBLEM FOR CATADIOPTRIC SENSORS: AN ANALYTICAL APPROACH.

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Abstract The perspective-N-point problem is a well known issue in computer vision. It consists in the determination of the distance between the camera and a set of points well known in an object coordinate space. This problem has been extensively treated in the literature and is still opened. Many solutions already exist. All these approaches consider only common planar camera. We propose, with a new formulation, to extend this problem to non linear imaging sensors: catadioptric panoramic sensors. The proposed approach permits to get a strictly analytical solution to the perspective-N-point problem usable with this kind of sensors.

Keywords: Pose estimation, perspective-N-point, catadioptric sensors

1. Introduction

Catadioptric sensors are a combination of a mirror and a video camera. A special and very interesting configuration is the one that uses a revolution profile mirror as it provides 360 field of view at a classical video frame rate (figure 1).

This topic is relatively new and most well mastered problems in planar cameras have to be extended for catadioptric sensors. The Perspective-N-point problem also called P-N-P (estimating the distance between the sensor and a set of N well known points) is one of these issues: many people have worked on it but it has never been used with catadioptric sensors. Multiple formu-

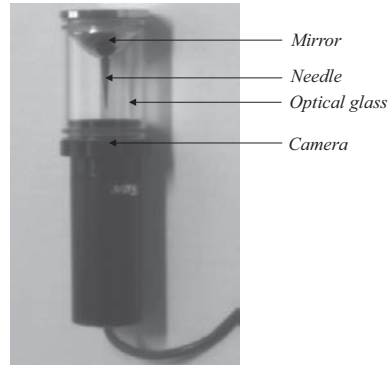


Figure 1. A panoramic catadioptric sensor from vstone (Vstone-Corporation,): a small camera is in front of a revolution profile mirror.

lations exist. The most popular is certainly the one exposed by Fischler and Bolles (Fischler and Bolles, 1981). The distance between the sensor and points is evaluated by the use of the cosines law in the perspective polyhedron. Linearization of this formulation is proposed by Quand and Lan in (Quan and Lan, 1999) and Ameller et al. in (Ameller et al., 2002). Haralick in (Haralick et al., 1991) reviews previous approach. Another formulation is the one presented by Horaud et al. (Horaud et al., 1989). They give an analytical solution to the perspective-4-point problem. Abidi and Chandra propose in (Abidi and Chandra, 1995) a solution for the P4P problem on a sensor that include a zoom control. A good overview of the PNP problem is given by Carceroni and Brown (Carceroni and Brown, 1997). They sum up most existing solutions. Perspective-N-point problem has been also treated under simplified model of camera like in (Alter, 1994): Alter gives a solution for the P3P problem under weak perspective.

All these formulations need a center of projection. Some catadioptric sensor have an optical center but not all of them. We need to find a new formulation if we want to be more general. We present a new method to solve the perspective-N-point problem and explain how to get an analytical solution. This method is used with catadioptric sensor and works, not only with sensors that respect the single viewpoint constraint, but with other kind of mirrors (and works also with planar cameras). The solution you get can be, in some cases complicated but, as the obtained solution is strictly analytical, the computational time is very low and constant which is well adapted for real time application. The accuracy is hard to quantify analytically as it depends on too many parameters like the resolution of the video camera, the size and the shape of the mirror, the precision of the localization of points in the image...

This paper is organized as followed: the first part explains the method, then an

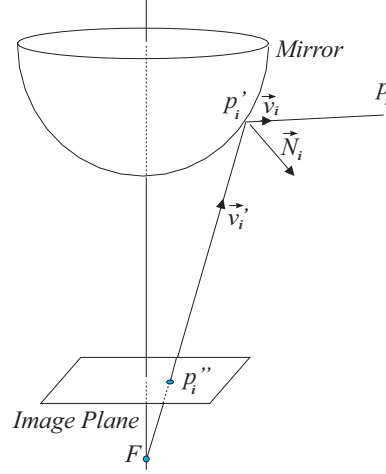


Figure 2. Projection of points p_i onto the mirror and onto the image.

example is given to illustrate the method. The second part shows how expressions are simplified if the sensor respects the single viewpoint constraint and the previous example is reviewed. Then comes the conclusion.

2. Solution of the P-N-P

The P-N-P is the problem of estimating the distance between the sensor and a well known set of points. Our sensor is supposed to be calibrated (Fabrizio et al., 2002) (Intrinsics parameters of the camera are known and relative placement of the mirror in respect to the camera is known too). As the transformation between the mirror coordinate space and the camera coordinate space is well known, we can then use indifferently one or the other. In our case, we will try to express the location of points in space in the mirror coordinate space.

Expression of the solution

Note p_i ($i = 1..N$) points in space, p'_i projection of points p_i on the mirror and p''_i projection of points p'_i on the image plan. Note F the center of the camera (figure 2). The issue is, given N points p''_i , find the location of points p_i in space.

With points p''_i , we can find points p'_i . Points p'_i are the intersection of line (Fp''_i) and the mirror. We can express the location of points p_i in space:

$$p_i = p'_i + \lambda_i \vec{v}_i \quad (1)$$

Note:

$$\lambda_i > 0 \quad (2)$$

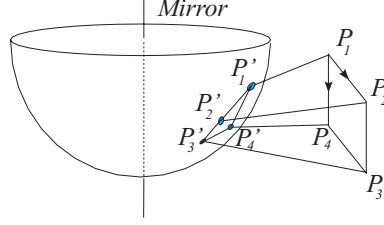


Figure 3. An example with 4 points.

with $\vec{v}_i$ directing vector of line $(p'_i p_i)$. The line $(p'_i p''_i)$ is the reflection of the line $(p'_i p_i)$, their directing vector are then linked: if we note $\vec{v}'_i$ the directing vector of line $(p'_i p''_i)$ and $\vec{N}_i$ the normal of the mirror in point p'_i , we can write:

$$\vec{v}_i = \vec{v}'_i - 2\vec{N}_i(\vec{N}_i \cdot \vec{v}'_i) \quad (3)$$

Note that all directing vectors are assumed to be unit.

To locate the N points p_i in space you have to find all λ_i , you have to find a system of equation with all λ_i and solve it. Note

$$\overrightarrow{p_i p_j} = p_j - p_i \quad (4)$$

$$= p'_j + \lambda_j \vec{v}_j - (p'_i + \lambda_i \vec{v}_i) \quad (5)$$

You can find the relation that expresses constraints on vector $\overrightarrow{p_i p_j}$. Solving the obtained system will give you analytical expression of all λ_i . Many constraints can be used like the length of vectors ($\|\overrightarrow{p_i p_j}\| = L_{ij}$). The simplest way to obtain systematically a solution is to express linear combination ($c_j \overrightarrow{p_i p_j} + c_k \overrightarrow{p_i p_k} = \overrightarrow{p_i p_l}$), and to solve the system to express all λ relatively to one of them. By the use of an additional equation ($\|\overrightarrow{p_i p_j}\| = L_{ij}$ for example), you can find the expression of the last λ . Note that, as points are well known in the object coordinate space, coefficients c and L_{ij} are known, the only unknown factors are the λ ... The complexity of the solution depends on expressions you choose. Once you get an expression for all λ , you get the distance between points in space and the center of the mirror then, you can estimate the transformation between the sensor coordinate space and the object coordinate space. This method allow you to get an analytical solution for $N > 3$ if the points are coplanar and for $N > 4$ otherwise and for $N = 3$ if the points are aligned.

Let us see on a simple example how to do.

Example

Suppose you have 4 points (p_1, p_2, p_3, p_4) coplanar in space and they form a parallelogram (figure 3). Their projection onto the mirror are noted p'_1, p'_2, p'_3

and p'_4 respectively and their projection on the image plan are noted p''_1, p''_2, p''_3 and p''_4 respectively. We can compute $\vec{v}_i$ as explained before and then compute vectors $\overrightarrow{p_1 p_2}$ and $\overrightarrow{p_3 p_4}$. As points are forming a parallelogram in space we can write:

$$\overrightarrow{p_1 p_2} + \overrightarrow{p_3 p_4} = 0 \quad (6)$$

this gives us the system:

$$\begin{bmatrix} p'_{x2} + \lambda_2 v_{x2} - (p'_{x1} + \lambda_1 v_{x1}) \\ + p'_{x4} + \lambda_4 v_{x4} - (p'_{x3} + \lambda_3 v_{x3}) \\ p'_{y2} + \lambda_2 v_{y2} - (p'_{y1} + \lambda_1 v_{y1}) \\ + p'_{y4} + \lambda_4 v_{y4} - (p'_{y3} + \lambda_3 v_{y3}) \\ p'_{z2} + \lambda_2 v_{z2} - (p'_{z1} + \lambda_1 v_{z1}) \\ + p'_{z4} + \lambda_4 v_{z4} - (p'_{z3} + \lambda_3 v_{z3}) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

rewritten differently:

$$\begin{bmatrix} \lambda_2 v_{x2} - \lambda_3 v_{x3} + \lambda_4 v_{x4} \\ \lambda_2 v_{y2} - \lambda_3 v_{y3} + \lambda_4 v_{y4} \\ \lambda_2 v_{z2} - \lambda_3 v_{z3} + \lambda_4 v_{z4} \end{bmatrix} = \begin{bmatrix} \lambda_1 v_{x1} \\ \lambda_1 v_{y1} \\ \lambda_1 v_{z1} \end{bmatrix} + \begin{bmatrix} v_{x0} \\ v_{y0} \\ v_{z0} \end{bmatrix} \quad (8)$$

with

$$v_0 = p'_1 - p'_2 + p'_3 - p'_4 \quad (9)$$

if we pose:

$$\Delta_{ijk} = \begin{vmatrix} v_{xi} & v_{xj} & v_{xk} \\ v_{yi} & v_{yj} & v_{yk} \\ v_{zi} & v_{zj} & v_{zk} \end{vmatrix} \quad (10)$$

the solution of the system (8) is

$$\lambda_2 = (\Delta_{340} + \Delta_{134}\lambda_1)/\Delta_{234} \quad (11)$$

$$\lambda_3 = (\Delta_{240} + \Delta_{124}\lambda_1)/\Delta_{234} \quad (12)$$

$$\lambda_4 = (\Delta_{230} + \Delta_{123}\lambda_1)/\Delta_{234} \quad (13)$$

We have an expression of all λ_i relatively one of them. All expressions are divided by Δ_{234} . Δ_{234} is never equal to zero because vector $\vec{v}_i$ are not collinear. There is no degenerated solution.

We have to find an expression of the last λ . If we pose L_{14} the length of the segment $[p_1 p_4]$, we can write:

$$\|\overrightarrow{p_1 p_4}\| = L_{14} \quad (14)$$

$$\Leftrightarrow \|p'_4 + \lambda_4 \vec{v}_4 - (p'_1 + \lambda_1 \vec{v}_1)\| = L_{14} \quad (15)$$

$$\begin{aligned} \Leftrightarrow \lambda_1^2 - 2\lambda_1\lambda_4\vec{v}_1\vec{v}_4 + 2(p'_1 - p'_4)(\lambda_1\vec{v}_1 - \lambda_4\vec{v}_4) \\ + \lambda_4^2 + p_1'^2 + p_4'^2 - 2p'_1 p'_4 = L_{14}^2 \\ (\text{with } \|\vec{v}_i\| = 1 \text{ and } L_{14} > 0) \end{aligned} \quad (16)$$

Combined with the expression of λ_1 (Eq. 13), we get:

$$\lambda_1^2 A + \lambda_1 B + C = 0 \text{ with } \lambda_1 > 0 \quad (17)$$

and

$$A = 1 + (\Delta_{123}/\Delta_{234})^2 - 2(\Delta_{123}/\Delta_{234})\vec{v}_1\vec{v}_4 \quad (18)$$

$$B = 2\Delta_{230}\Delta_{123}/\Delta_{234}^2 + 2(p'_1 - p'_4)(\vec{v}_1 - \Delta_{123}/\Delta_{234}\vec{v}_4) - 2\Delta_{230}/\Delta_{234}\vec{v}_1\vec{v}_4 \quad (19)$$

$$C = -2(p'_1 - p'_4)\Delta_{230}/\Delta_{234}\vec{v}_4 + (p'_1 - p'_4)^2 + (\Delta_{230}/\Delta_{234})^2 - l^2 = 0 \quad (20)$$

This polynomial expression (Eq. (17)) can be easily solved and give an expression of λ_1 . This solution, combined with Eqs. (11) to (13), gives the expression of all λ_i . Note that Eq. (17) has two solutions but the correct one is the positive one.

We have explained how to get an analytical solution of the P-N-P, then we have given an example of simple solution with $N = 4$ and shape constrained. This constraint was added to simplify the expression of the solution and to make it shorter. If you want to get the general solution (without any constraints on shape), you just have to change equation (6) by

$$K_1 \vec{p_1 p_2} + K_2 \vec{p_1 p_4} - \vec{p_1 p_3} = 0 \quad (21)$$

Now let us see simplifications that bring the single viewpoint constraint.

3. Simplification of the solution with the single viewpoint constraint.

The single viewpoint constraint ensures the presence and the uniqueness of the optical center of the mirror (Baker and Nayar, 1999). The computation of reflection on the mirror is simplified and particularly we are not obliged to compute the normal of the mirror to compute the reflection (figure 4). If we note F' the optical center of the mirror, the Eq. (1) can be rewritten:

$$p_i = F' + \lambda_i \vec{v}_i \quad (22)$$

with

$$\vec{v}_i = (p'_i - F')/(||p'_i - F'||) \quad (23)$$

Note that points p'_i are always known as they are at the intersection of line (Fp''_i) and the mirror. Expression of vectors $\vec{p_i p_j}$ (Eq. (5)) is then simplified:

$$\vec{p_i p_j} = F' + \lambda_j \vec{v}_j - (F' + \lambda_i \vec{v}_i) \quad (24)$$

$$= \lambda_j \vec{v}_j - \lambda_i \vec{v}_i \quad (25)$$

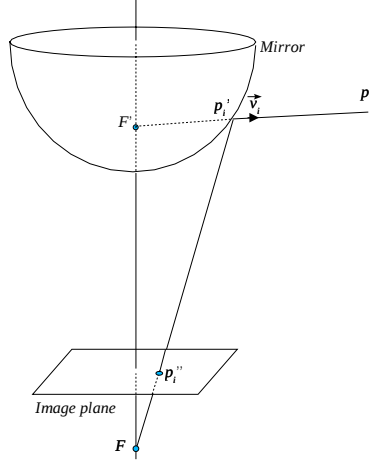


Figure 4. Projection of points p_i according to the optical center of the mirror.

If the preceding example with the parallelogram $p_1p_2p_3p_4$ is taken again, system (8) becomes:

$$\begin{bmatrix} \lambda_2 v_{x2} + \lambda_4 v_{x4} - \lambda_3 v_{x3} \\ \lambda_2 v_{y2} + \lambda_4 v_{y4} - \lambda_3 v_{y3} \\ \lambda_2 v_{z2} + \lambda_4 v_{z4} - \lambda_3 v_{z3} \end{bmatrix} = \begin{bmatrix} \lambda_1 v_{x1} \\ \lambda_1 v_{y1} \\ \lambda_1 v_{z1} \end{bmatrix} \quad (26)$$

the solution of this system is then:

$$\lambda_2 = (\Delta_{134}/\Delta_{234}) \lambda_1, \quad (27)$$

$$\lambda_3 = (\Delta_{124}/\Delta_{234}) \lambda_1, \quad (28)$$

$$\lambda_4 = (\Delta_{123}/\Delta_{234}) \lambda_1 \quad (29)$$

In the same way, Eq. (14) becomes:

$$\begin{aligned} \|\overrightarrow{p_1 p_4}\| &= L_{14} \\ \Leftrightarrow \|\lambda_4 \vec{v}_4 - \lambda_1 \vec{v}_1\| &= L_{14} \end{aligned} \quad (30)$$

$$\begin{aligned} \Leftrightarrow \lambda_4^2 - 2\vec{v}_1 \vec{v}_4 \lambda_1 \lambda_4 + \lambda_1^2 &= L_{14}^2 \\ (\text{because } \|\vec{v}_i\| &= 1 \text{ and } L_{14} > 0) \end{aligned} \quad (31)$$

combined with new solution (29) we get:

$$\lambda_1^2 = L_{14}^2 / \delta \quad (32)$$

$$\delta = 1 - 2\vec{v}_1 \vec{v}_4 \Delta_{123} / \Delta_{234} + \Delta_{123}^2 / \Delta_{234}^2 \quad (33)$$

If we introduce this solution in Eqs. (27)-(29), we get a solution for each λ :

$$\lambda_1 = L_{14} / \sqrt{\delta}, \quad (34)$$

$$\lambda_2 = (\Delta_{134} L_{14})/(\Delta_{234} \sqrt{\delta}), \quad (35)$$

$$\lambda_3 = (\Delta_{124})/(\Delta_{123}) (L_{14})/(\sqrt{\delta}), \quad (36)$$

$$\lambda_4 = (\Delta_{234})/(\Delta_{234}) (L_{14})/(\sqrt{\delta}) \quad (37)$$

The solution is then drastically simplified. (Note that this solution also works with planar camera).

4. Conclusion

We have presented a new way to solve the perspective-N-point problem. The solution is strictly analytical and works well with every kind of catadioptric panoramic sensors (but is not restricted to them). Expressions of solutions are simplified when the sensor respects the single viewpoint constraint. The choice of the geometrical form of points p_i makes it possible to get a simple solution. This solution has a constant execution time. The solution can be used directly or can be used to initialize a minimisation process.

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